Model Predictive Functional Control for Processes with Unstable Poles

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Abstract

In this paper the coprime-factorized model predictive functional control for single-input single-output processes with an arbitrary number of unstable poles is presented. The predictive functional control algorithm gives a framework for designing the control for a wide range of processes. The main idea in the case of unstable poles is based on the prediction of the process output based on the coprime-factorized process model. The robust stability of the proposed control algorithm is also discussed, using the small-gain theorem, which provides a sufficient condition for stability.

Keywords: unstable processes; predictive control;

1 Introduction

In real systems it is possible that multiple equilibrium points exist due to the degree of nonlinearity, and sometimes some of these equilibrium points are unstable. The control of such systems is especially difficult when they have more unstable modes, this is mainly because of the difficulty in ensuring the robustness for the whole control system, as reported in Razon and Schmitz (1987).

Several different approaches exist for solving the problem of open-loop unstable process control. A review of the methods, mainly based on classical PID controllers, is given by Chidambaram (1997). Another solid framework for designing the control of unstable processes is the internal model control strategy, which is discussed by DePaor (1985) and Kaya (2004). The most important analytical framework for developing the control of open-loop unstable processes is the robust control design framework, which also ensures robust control performance in the event of model uncertainty (Morari and Zafiriou, 1989), (Doyle *et al.*, 1990). In this framework several results were published using the stable factorization approach given by Vidyasagar (1985). The extension of the factorization approach to decentralized control by introducing decentralized stable factors, which allows the parametrization of the set of all the decentralized controllers that could stabilize the closed-loop system, is given by Date and Chow (1994), and the framework for the robust decentralized controller design of unstable systems is given by Loh and Chiu (1997).

The methods of classical model-based predictive control are not suitable for use with unstable plants (Muske and Rawlings, 1993); this implies there is a limitation on the performance that can be achieved using these controllers due to the structural error in the process model. The model-based predictive control methods that are based on a description in the state-space domain or by transfer functions, such as generalized predictive control and receding-horizon tracking control, are able to deal with unstable plants. The linear model predictive control of unstable processes is given by Muske and Rawlings (1993). The control is based on a quadratic performance criterion subject to the input and state constraints. The model predictive formulation to control the open-loop unstable processes is reported by Nagrath *et al.* (2002) as an optimization control problem. Some ideas for combining the robust design approaches and the generalized predictive control algorithm are discussed by Banerjee and Shah (1995). A priori stability conditions for an arbitrary number of unstable poles is given by Kouvaritakis *et al.* (1996).

The predictive functional control framework proposed by Richalet *et al.* (1978) can be used for a wide range of different processes, even in the case of multivariable processes, as proposed in Škrjanc *et al.* (2004). To extend the proposed framework to unstable processes the coprime factorization of the process model is proposed. In this paper we propose an algorithm for single-input single-output systems (SISO).

The paper is organized in the following way: Section 2 introduces coprime-factorized model predictive functional control (CFMPFC), Section 3 discusses the robust stability of CFMPFC, and Section 4 describes a simulation study of CFMPFC for processes with multiple unstable modes.

2 CFMPFC

In this section the coprime-factorized model predictive functional control will be introduced. The SISO model with r unstable and $n-r$ stable poles is described by the following discrete-time transfer function

$$G(z) = \frac{B(z)}{A^-(z)A^+(z)} \tag{1}$$

where $A^-(z)$ stands for a polynomial with $n-r$ stable poles and $A^+(z)$ stands for a polynomial with r unstable poles. The polynomial $A^+(z)$ is denoted as

$$A^+(z) = z^r + a_1^+ z^{r-1} + \cdots + a_{r-1}^+ z + a_r^+ \tag{2}$$

In general, it is always possible to find a coprime factorization of certain transfer functions, i.e., it is always possible to find two interconnected transfer functions with no unstable pole-zero cancellations (Glover and McFarlane, 1989), (Maciejowski, 2002). This means that it is possible to factorize the unstable part of the process transfer function into the feedback interconnection of two stable systems, as given in Eq. 3.

$$\frac{1}{A^+(z)} = \frac{1}{\mathcal{A}(z)} \frac{1}{1 - \frac{\mathcal{B}(z)}{\mathcal{A}(z)}} \tag{3}$$

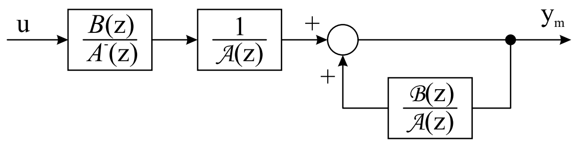

Figure 1: The decomposed model.

where $\mathcal{A}(z)$ stands for a stable polynomial of order r defined by the designer

$$\mathcal{A}(z) = z^r + \alpha_1 z^{r-1} + \cdots + \alpha_{r-1} z + \alpha_r \tag{4}$$

The poles of the polynomial affect the performance and the robustness of the control system, which will be discussed later. The numerator of the feedback transfer function equals

$$\mathcal{B}(z) = \beta_1 z^{r-1} + \cdots + \beta_{r-1} z + \beta_r$$

where the coefficients of $\mathcal{B}(z)$ are calculated to fulfill Eq. 3. This means that the coefficient of the polynomial $\mathcal{B}(z)$ should equal

$$\beta_i = \alpha_i - a_i^+, \; i = 1, \ldots, r \tag{5}$$

Fig. 1 shows the coprime-factorization of an unstable process model. The calculation of the model output, taking into account the decomposition in Fig. 1, results in

$$Y_m(k) = G_{m_1}(z)U(z) + G_{m_2}(z)Y_m(z) \tag{6}$$

where $Y_m(z)$ and $U(z)$ are Z-transforms corresponding discrete variables, and $G_{m_1}(z)$ and $G_{m_2}(z)$ stand for

$$G_{m_1}(z) = \frac{B(z)}{A^-(z)\mathcal{A}(z)}, \quad G_{m_2}(z) = \frac{\mathcal{B}(z)}{\mathcal{A}(z)} \tag{7}$$

The predictor proposed in Eq. 6 is inappropriate when the unstable process output is forecasted. Taking into account the assumption of the equivalence between the model, $y_m(k)$, and process output, $y_p(k)$, the suitable predictor model is obtained and is written as

$$Y_m(k) = G_{m_1}(z)U(z) + G_{m_2}(z)Y_p(z) \tag{8}$$

where $Y_p(z)$ stands for the Z-transform of the process output, $y_p(k)$. The scheme of the coprime-factorized predictor is presented in Fig. 2, where $G_p(z)$ stands for an accurate representation of the real plant. The prediction of the process model output is, therefore, composed of the prediction based on the input signal, $u(k)$, and the prediction based on the output process signal, $y_p(k)$. The H-step-ahead prediction of the model output is then calculated in the state-space domain. The decomposed process model transfer functions, $G_{m_1}(z)$ and $G_{m_2}(z)$ are based on the assumption of observability, transformed into

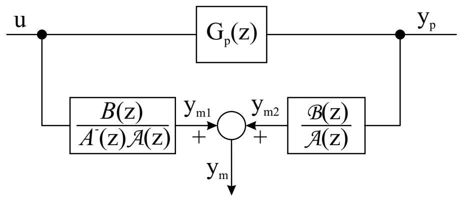

Figure 2: The coprime-factorized predictor.

the observable canonical state-space form. The first transfer function, $G_{m_1}(z)$, is represented with the state-space matrices $A_{m_1} \in \mathbb{R}^{n\times n}, B_{m_1} \in \mathbb{R}^{n\times 1}, C_{m_1} \in \mathbb{R}^{1\times n}$ and the second transfer function, $G_{m_2}(z)$, with $A_{m_2} \in \mathbb{R}^{r\times r}, B_{m_2} \in \mathbb{R}^{r\times 1}, C_{m_2} \in \mathbb{R}^{1\times r}$, assuming the dynamics where both input-output matrices, $D_1 \in \mathbb{R}$ and $D_2 \in \mathbb{R}$, are equal to zero. The outputs of the first and the second decomposed systems are now written in the state-space domain as follows:

$$\begin{aligned} x_{m_1}(k+1) &= A_{m_1}x_{m_1}(k) + B_{m_1}u(k), \quad y_{m_1}(k) = C_{m_1}x_{m_1}(k) \\ x_{m_2}(k+1) &= A_{m_2}x_{m_2}(k) + B_{m_2}y_p(k), \quad y_{m_2}(k) = C_{m_2}x_{m_2}(k) \end{aligned} \tag{9}$$

where $x_{m_1}(k)$ and $x_{m_2}(k)$ stand for the model states. The prediction of the model output is the sum of the prediction to both decomposed systems, written as follows:

$$\begin{aligned} y_m(k+H) &= y_{m_1}(k+H) + y_{m_2}(k+H) \\ y_{m_1}(k+H) &= C_{m_1}\left(A_{m_1}^H x_{m_1}(k) + \sum_{i=1}^{H} A_{m_1}^{H-i}B_{m_1}u(k+i-1)\right) \\ y_{m_2}(k+H) &= C_{m_2}\left(A_{m_2}^H x_{m_2}(k) + \sum_{i=1}^{H} A_{m_2}^{H-i}B_{m_2}y_p(k+i-1)\right) \end{aligned} \tag{10}$$

The main idea of the predictive functional control is given in Eq. 11

$$w(k+H) - y_m(k+H) = a_r^H e(k) \tag{11}$$

where $w(k+H)$ is the H-step-ahead reference signal, $y_m(k+H)$ is the H-step-ahead prediction of the process model output, and a_r is the exponential factor ($0 < a_r < 1$), which introduces an exponentially decreasing control error $e(k) = w(k) - y_p(k)$ and defines the behavior of the closed-loop system, as follows from Škrjanc and Matko (2000).

The H-step-ahead prediction in Eq. 10 assumes a constant input variable (mean level predictive control) for the whole prediction horizon ($u(k) = u(k+1) = \ldots = u(k+H-1)$), assuming a constant reference variable $w(k)$ for the whole prediction horizon ($w(k) = w(k+1) = \ldots = w(k+H)$) and assuming that the model output, $y_m(k)$, approximately follows the process output, $y_p(k)$, which means that the term $y_p(k+i)$ in Eq. 10 equals $y_p(k+i) = w(k+i) - a_r^i e(k)$, for $i = 1, \cdots, H$, written in the following way

$$y_m(k+H) = C_{m_1} A_{m_1}^H x_{m_1}(k) + \Gamma_1 u(k) + C_{m_2} A_{m_2}^H x_{m_2}(k) + \Gamma_{21} w(k) - \Gamma_{22} e(k) \tag{12}$$

where the matrices Γ_1, Γ_{21} and Γ_{22} are as follows

$$\Gamma_1 = C_{m_1} \left(A_{m_1}^H - I_1 \right) \left(A_{m_1} - I_1 \right)^{-1} B_{m_1}$$

$$\Gamma_{21} = C_{m_2} \left(A_{m_2}^H - I_2 \right) \left(A_{m_2} - I_2 \right)^{-1} B_{m_2}$$

$$\Gamma_{22} = C_{m_2} \left(A_{m_2}^H - a_r^H I_2 \right) \left(A_{m_2} - a_r I_2 \right)^{-1} B_{m_2}$$

and $I_1 \in \mathbb{R}^{n \times n}$ and $I_2 \in \mathbb{R}^{r \times r}$ stand for the unity matrices.

Introducing Eq. 12 into Eq. 11, the following is obtained

$$w(k+H) - \left(C_{m_1} A_{m_1}^H x_{m_1}(k) + \Gamma_1 u(k) + C_{m_2} A_{m_2}^H x_{m_2}(k) + \Gamma_{21} w(k) - \Gamma_{22} e(k) \right) = a_r^H e(k) \tag{13}$$

Solving Eq. 13 for the variable $u(k)$, the control law of CFMPFC is obtained as follows

$$u(k) = g \left(w(k) - y_p(k) \right) + K_{m_1} x_{m_1}(k) + K_{m_2} x_{m_2}(k) + K_{y_p} y_p(k) \tag{14}$$

where

$$\begin{aligned} g &= g_0^{-1} \left(1 - a_r^H + \Gamma_{22} - \Gamma_{21} \right) \\ g_0 &= \Gamma_1 \\ K_{m_1} &= g_0^{-1} C_{m_1} \left(I_1 - A_{m_1}^H \right) \\ K_{m_2} &= g_0^{-1} C_{m_2} \left(I_2 - A_{m_2}^H \right) \\ K_{y_p} &= -g_0^{-1} \Gamma_{21} \end{aligned} \tag{15}$$

Note that the control law from Eq. 15 is realizable if the gain g_0 is non-zero. This is true if $H \geq \rho$, where ρ is the relative order of the system, as shown in Škrjanc *et al.* (2004). In the case of constraints of the process variables, they can be taken into account inside the inner process model. The control signal, for example, can be stripped out and in parallel led to the process input and to the inner process model.

2.1 Integral nature of CFMPFC

For the subsequent investigation, the basic control scheme will be rearranged into the scheme in Fig. 3, where

$$\begin{aligned} F(z) &= K_{m_1} \left(zI_1 - A_{m_1} \right)^{-1} B_{m_1} \\ H(z) &= K_{m_2} \left(zI_2 - A_{m_2} \right)^{-1} B_{m_2} + K_{y_p} \end{aligned} \tag{16}$$

A very important feature of all control algorithms is their behavior at low frequencies. The control

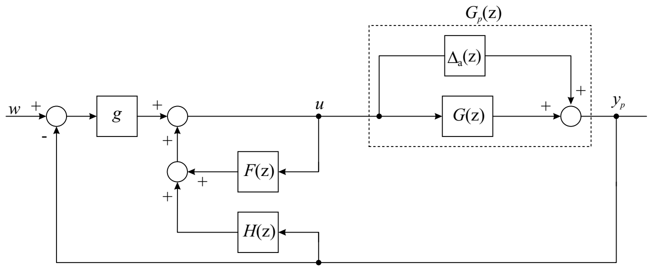

Figure 3: The control scheme of CFMPFC.

algorithm should be able to suppress the control error in the steady-state, i.e., the control law should have an integral nature.

The control law of CFMPFC in the $\mathcal{Z}$-*domain* is written as:

$$U(z) = (1 - F(z))^{-1} (gE(z) + H(z)Y_p(z)) \tag{17}$$

where $E(z)$ and $Y_p(z)$ stand for the $\mathcal{Z}$-*transforms* of the control error $e(k)$ and the output variable $y_p(k)$, respectively. To prove the integral nature we have to investigate the behavior of the control law at low frequencies, i.e., we have to show that the transfer function between the control error and the control variable has a pole at $z = 1$. By calculating the denominator of the transfer functions in Eq. 17 at $z = 1$ and by taking into account Eq. 15 and Eq. 16 the following is obtained:

$$1 - F(1) = 1 - K_{m_1} (zI_1 - A_{m_1})^{-1} B_{m_1} \tag{18}$$

By taking into account Eq. 15 it is obvious that:

$$1 - F(1) = 1 - g_0^{-1} g_0 = 0 \tag{19}$$

This proves that the control law of CFMPFC is indeed integral in nature, i.e., y_p asymptotically tracks a step reference signal.

3 Robust stability

The internal stability of the control system with the nominal plant $G(z)$ (a plant without uncertainty, $G_p(z) \equiv G(z)$) can be investigated by studying the roots of the closed-loop denominator, which is given as $P_{cl}(z) = 1 - F(z) + G(z)(g - H(z))$. The performance and robustness of model-based control schemes depend mainly on the uncertainty between the model and the plant. In the case of a stabilization problem, the most important factor is the robust stability of the closed-loop system. One of the most important tools for investigating the stability of the system in the presence of model-plant uncertainty is the small-gain theorem, as discussed by Doyle *et al.* (1990). The small-gain theorem provides a sufficient condition for the stability of the control system. This means that a violation of the small-gain stability criteria, even with an exact knowledge of the uncertainty, may or may not lead to instability (Morari and Zafiriou, 1989).

Additive perturbation, coprime-factor perturbation, multiplicative perturbation at the control input and multiplicative perturbation at the control output, are the main forms of unstructured uncertainties in linear systems. In fact all these uncertainties can be presented by additive perturbations, as shown by Wang (1997).

Therefore, in our case the uncertainty between the actual plant $G_p(z)$ and the plant model $G(z)$ will be described by an additive unstructured uncertainty

$$G_p(z) = G(z) + \Delta_a(z) \tag{20}$$

The structure of $\Delta_a(z)$ is usually unknown but stable, and it is an upper-bounded function in the frequency domain

$$|\Delta_a(e^{-j\omega T})| < \delta_a(\omega), \ \ \forall \ \ \omega T \in [0, \pi] \tag{21}$$

where T stands for the sampling time. The upper bound $\delta_a(\omega)$ can be approximated from experiment.

Taking into account Eq. 20 and Eq. 21, the family of plants, $\mathcal{G}_a$, can be described by

$$\mathcal{G}_a = \{G_p \ : \ \ |G_p(e^{-j\omega T}) - G(e^{-j\omega T})| < \delta_a(\omega), \ \ \forall \ \ \omega T \in [0, \pi]\} \tag{22}$$

In Fig. 3 the coprime-factorized model predictive functional control scheme is presented, where Δ_a represents the additive unstructured uncertainty and $g, F(z)$ and $H(z)$ stand for the controller that stabilizes the nominal plant. The controller can also be denoted by the triplet (g, F, H). A rearrangement of the system in Fig. 3, where all the external inputs and outputs are neglected, results in a general M-Δ interconnection structure (Fig. 4), as defined by Morari and Zafiriou (1989). The interconnection matrix

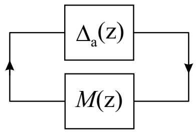

Figure 4: The interconnection structure.

M derived from Fig. 3 equals

$$M(z) = \frac{-g + H(z)}{1 - F(z) - G(z)H(z) + gG(z)} \tag{23}$$

Lemma 1 (Robust stability and the robust stability margin for additive perturbation). *Assuming that all plants $G_p(z)$ in the family $\mathcal{G}_a$ defined in Eq. 22 have the same number of unstable poles (outside the unity circle in the z-plane) and that the controller (g, F, H) internally stabilizes the nominal plant G, then the system is robustly stable if and only if*

$$\|\delta_a(z)M(z)\|_\infty \triangleq \sup_\omega |M(e^{-j\omega T})\delta_a(e^{-j\omega T})| < 1 \tag{24}$$

or in more conservative form:

$$|M(e^{-j\omega T})| \cdot |\delta_a(e^{-j\omega T})| < 1, \;\; \forall\, \omega T \in [0, \pi] \tag{25}$$

If we define the upper robust stability margin $\overline{\delta}_a(\omega)$ as

$$\overline{\delta}_a(\omega) \triangleq \frac{1}{|M(e^{-j\omega T})|}, \;\; \omega T \in [0, \pi] \tag{26}$$

then the system is robustly stable if and only if

$$|\delta_a(e^{-j\omega T})| < \overline{\delta}_a(\omega), \;\; \forall\, \omega T \in [0, \pi] \tag{27}$$

Lemma 1 is the result of applying the small-gain theorem to the interconnection structure from Fig. 4. Taking into account Lemma 1 and Eq. 23 the following theorem for the robust stability of CFMPFC is obtained.

Theorem 1 (Robust stability of CFMPFC for additive perturbation). *Assuming that all plants $G_p(z)$ in the family $\mathcal{G}_a$*

$$\begin{aligned}\mathcal{G}_a = \{G_p: \;\; &|G_p(e^{-j\omega T}) - G(e^{-j\omega T})| < \delta_a(\omega), \\ &\forall\, \omega T \in [0, \pi]\}\end{aligned} \tag{28}$$

have the same number of unstable poles, and that the controller $(g, F(z), H(z))$ internally stabilizes the nominal plant $G(z)$, then the system is robustly stable if and only if

$$\left| \frac{-g + H(e^{-j\omega T})}{1 - F(e^{-j\omega T}) - G(e^{-j\omega T})\,(-g + H(e^{-j\omega T}))} \delta_a(e^{-j\omega T}) \right| < 1 \tag{29}$$

or in more conservative form:

$$\begin{aligned}\left| \frac{-g + H(e^{-j\omega T})}{1 - F(e^{-j\omega T}) - G(e^{-j\omega T})\,(-g + H(e^{-j\omega T}))} \right| \cdot \left|\delta_a(e^{-j\omega T})\right| < 1 \\ \forall\, \omega T \in [0, \pi]\end{aligned} \tag{30}$$

Remark 1 (Robust upper-stability margin of CFMPFC for additive perturbation). *The robust upper-stability margin $\overline{\delta}_a(\omega)$ of CFMPFC for additive unstructured perturbations is defined as follows:*

$$\overline{\delta}_a(\omega) \triangleq \left| \frac{1 - F(e^{-j\omega T}) - G(e^{-j\omega T})\left(-g + H(e^{-j\omega T})\right)}{-g + H(e^{-j\omega T})} \right|, \ \ \omega T \in [0, \pi] \tag{31}$$

According to the robust upper-stability margin the system is robustly stable if and only if

$$\left|\delta_a(e^{-j\omega T})\right| < \overline{\delta}_a(\omega), \ \ \forall\, \omega T \in [0, \pi] \tag{32}$$

4 Simulation study

The approach to the tuning of CFMPFC presented here is based on shaping the frequency-domain robust-stability margins. The approach also gives an insight into the role of the tuning parameters and its influence on stability. The simulation study was carried out on a discrete transfer function with two unstable and one stable mode, given as follows:

$$G(z) = \frac{B(z)}{A(z)} = \frac{0.0312z^2}{(z - 1.0126)\,(z - 1.0050)\,(z - 0.9512)} \tag{33}$$

with the sampling time $T = 0.05s$. The unstable polynomial $A^+(z)$ is then

$$A^+(z) = z^2 + a_1^+ z + a_2^+ = z^2 - 2.0176z + 1.0177 \tag{34}$$

The polynomial $\mathcal{A}(z)$ has the main effect on the robustness of the whole system. This design polynomial is of the order r and its coefficients should be defined by the designer. The polynomial is defined as follows

$$\mathcal{A}(z) = z^2 + \alpha_1 z + \alpha_2 = (z - \lambda)^r \tag{35}$$

This means that the design of the polynomial $\mathcal{A}(z)$ is reduced to only one parameter (λ). By taking into account Eq. 5 and Eq. 35 the coefficients of the polynomial $\mathcal{B}(z)$ in our example equal

$$\beta_1 = -2\lambda + 2.0176 \tag{36}$$

$$\beta_2 = \lambda^2 - 1.0177 \tag{37}$$

The robust-stability margins depend mainly on the parameters λ and a_r. In Fig. 5 the robust-stability margins are shown for different values of the parameter λ, where $a_r = 0.9959$ and $H = 3$. It is shown that the robust stability at higher frequencies increases with decreasing λ, but at lower frequencies λ has almost no influence on the robust stability. The robust stability is discussed for the case of the model mismatch given by the additive uncertainty

$$\Delta_a(z) = \frac{-0.025z + 0.0336}{(z - 0.1353)A(z)}$$

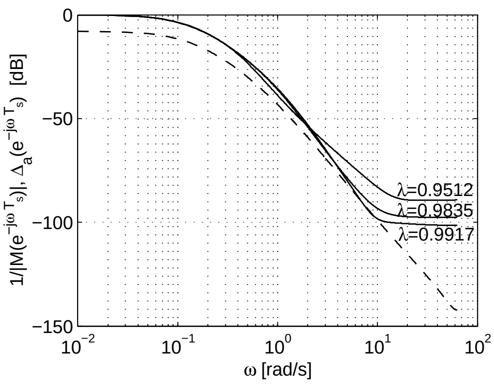

Figure 5: The robust-stability margins for different values of λ.

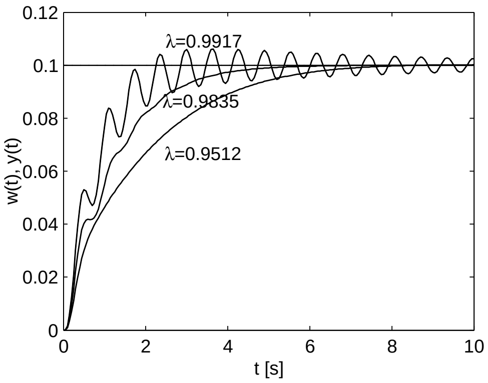

Figure 6: The output signal for the case of different λ .

This uncertainty involves a parasitic high-frequency pole and uncertainty at low frequencies. In Fig. 6 the closed-loop system responses in the case of a different λ ($a_r = 0.9959, H = 3$) parameter and assuming an additive uncertainty are shown. In Fig. 7 the robust-stability margins are shown for different values of the parameter a_r, where $\lambda = 0.9835, H = 3$. It is shown that the robust stability at higher frequencies increases with increasing a_r, but at lower frequencies a_r has almost no influence on the robust stability.

5 Conclusion

In this paper the coprime-factorized model predictive functional control is given for single-input single-output systems with multiple unstable modes. The proposed approach is an extension of the well-known predictive functional control, which can be used to control a wide range of different processes, to the unstable systems. The robust stability of the proposed control algorithm is also discussed using the small-gain theorem, which provides a sufficient condition for the stability of the control system.

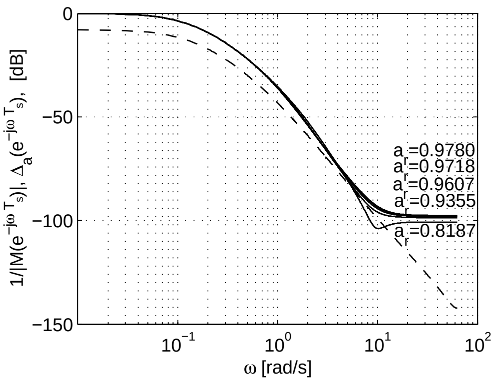

Figure 7: The robust-stability margins for different values of a_r .

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